Question 9.

Geometric distribution: $P(x) = q^{x-1}p$

$p = \frac{1}{10}$ and $q = \frac{9}{10}$.

(a) $P(X \leq 4) = \sum_{x=1}^{4}\left(q^{(x-1)}p\right) = \sum_{x=1}^{4}\left(\frac{9}{10}^{(x-1)} \cdot \frac{1}{10}\right) = \frac{1}{10}\sum_{x=1}^{4}\frac{9}{10}^{(x-1)}$

$= \frac{1}{10}\left\{\left(\frac{9}{10}\right)^0 + \left(\frac{9}{10}\right)^1 + \left(\frac{9}{10}\right)^2 + \left(\frac{9}{10}\right)^3\right\}$ (x=1, x=2, x=3)

$= \frac{1}{10}\left\{1 + \frac{9}{10} + \frac{81}{100} + \frac{729}{1000}\right\} = 0.3439$

(b) $P(X \leq 4 \mid X \geq 2)$.

By defn, $P(B|A) = \frac{P(A) \cap P(B)}{P(A)}$

$\therefore P(X \geq 2) = 1 - P(X < 2) = 1 - P(x = 1)$

But $P(X=1) = P(1) = \frac{9}{10}^{(1-1)} \cdot \frac{1}{10} = 1 \times \frac{1}{10} = \frac{1}{10} = 0.1$

Assuming that the events are independent,

$\therefore P(X \leq 4 \mid X \geq 2) = \frac{P(X \leq 4) \cdot P(X \geq 2)}{P(X \geq 2)}$

$= \frac{0.3439 \times 0.1}{0.1}$

$= 0.3439.$